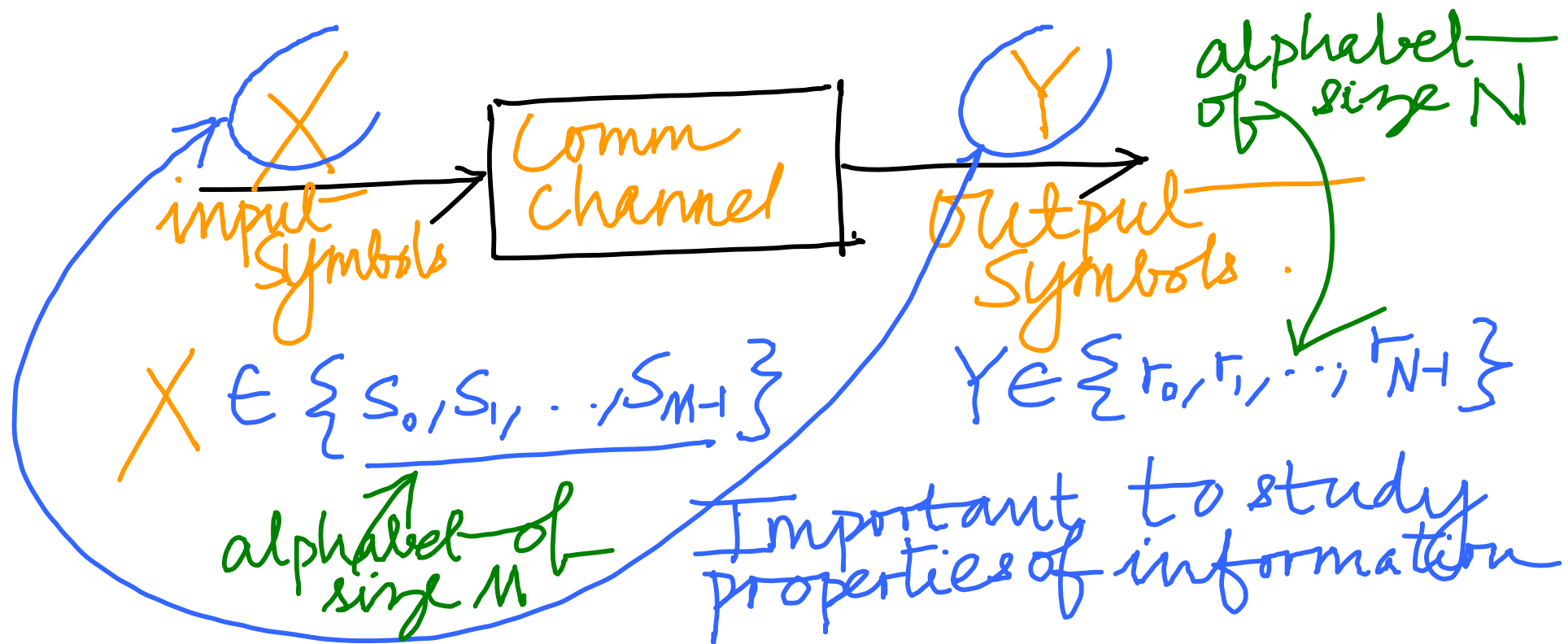


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JOINT ENTROPY

Combined Information



in X, Y to understand
fundamental limits
of communication.

$$\begin{matrix} X & = & s_i \\ Y & = & r_j \end{matrix}$$

$X \backslash Y$	r_0	r_1	...	...	r_{N-1}
s_0					
s_1					
$\vdots$					
s_{M-1}					

P_{ij}

$$P_{ij} = \Pr(X = s_i, Y = r_j)$$

$$P_{ij} = \Pr(X = s_i \cap Y = r_j)$$

$$0 \leq i \leq M-1$$

$$0 \leq j \leq N-1$$

Naturally, one can think
of (X, Y) as a 2-Dimensional
source with symbols (s_i, r_j)

Total # such
2D symbols = MN .

occurs with
probability P_{ij}

Joint Entropy $H(X, Y)$

$$H(X, Y) = \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} P_{ij} \log_2 \left(\frac{1}{P_{ij}} \right)$$

Joint Entropy

Information conveyed
by X, Y together

Example:

$$Pr(X = s_0) = \frac{1}{4}$$

$$\begin{aligned} Pr(r_0) &= \frac{1}{8} + \frac{1}{16} \\ &+ \frac{1}{16} + \frac{1}{4} \\ &= \frac{1}{2} \end{aligned}$$

All are equiprobable.

$X \backslash Y$		$\frac{1}{2}$ r_0	$\frac{1}{4}$ r_1	$\frac{1}{8}$ r_2	$\frac{1}{8}$ r_3
$\frac{1}{4}$ s_0	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{32}$	
$\frac{1}{4}$ s_1	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$	
$\frac{1}{4}$ s_2	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	
$\frac{1}{4}$ s_3	$\frac{1}{4}$	0	0	0	

$$\frac{1}{32} = Pr(X = s_1, Y = r_2)$$

$$\frac{1}{8} = p_{12}$$

$$Pr(Y = r_3) = \frac{1}{8}$$

$$\begin{aligned} \text{Total \# 2D symbols} &= 4 \times 4 \\ &= 16. \end{aligned}$$

Check: ←

check is probability
Distribution is valid,

$$\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} P_{ij} = 1$$

Total Probability

Total Probability

$$= \frac{1}{4} + 2 \times \frac{1}{8} + 6 \times \frac{1}{16}$$

$$+ 4 \times \frac{1}{32} \quad \checkmark$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{3}{8} + \frac{1}{8} = 1$$

↑ valid Probability
Distribution

Total Probability Rule

$$\sum_{i=0}^{M-1} \Pr(s_i, r_j) = \Pr(r_j)$$

disjoint

$s_0, s_1, \dots, s_{M-1}$

Mutually Exclusive & Exhaustive.

$$\Pr(s_i, r_j) = \Pr(s_i \cap r_j)$$

$$\Pr(A_0 \cap B) + \Pr(A_1 \cap B) + \dots + \Pr(A_{M-1} \cap B) = 1$$

$$\bigvee A_i \cap A_j = \emptyset \quad \forall i, j$$

$A_0, A_1, \dots, A_{n-1}$ are mutually Exclusive

$$A_0 \cup A_1 \cup \dots \cup A_{n-1} = S$$

Sample Space.

$$\sum_{j=0}^{N-1} \Pr(s_i, r_j) = \Pr(s_i)$$

Joint Entropy (X, Y) .

$$H(X, Y) = \sum_i \sum_j P_{ij} \log_2 \left(\frac{1}{P_{ij}} \right)$$

$$= 2 \times \frac{1}{8} \log_2 8 + 6 \times \frac{1}{16} \log_2 16$$

$$+ \frac{1}{4} \log_2 4 + \frac{4}{32} \log_2 32$$

$$= \frac{3}{4} + \frac{3}{2} + \frac{1}{2} + \frac{5}{8}$$

$$= \frac{6 + 12 + 4 + 5}{8} = \frac{27}{8} = 3.375 \text{ bits.}$$

$$H(X, Y) = 3.375_{\text{bits}}$$



Treating symbols
as 2D symbols.